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NON-ANTICOMMUTATIVE DEFORMATION OF N=(1,1) HYPERMULTIPLETS

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Abstract

We study the SO(4)×SU(2) invariant and $N{=}(1,0)$ supersymmetry-preserving nilpotent (non-anticommutative) Moyal deformation of hypermultiplets interacting with an abelian gauge multiplet, starting from their off-shell formulation in Euclidean $N{=}(1,1)$ harmonic superspace. The deformed version of a neutral or a charged hypermultiplet corresponds to the 'adjoint' or the 'fundamental' representation of the deformed U(1) gauge group on the superfields involved. The neutral hypermultiplet action is invariant under $N{=}(2,0)$ supersymmetry and describes a deformed $N{=}(2,2)$ gauge theory. For both the neutral and the charged hypermultiplet we present the corresponding component actions and explicitly give the Seiberg-Witten-type transformations to the undeformed component fields. Mass terms and scalar potentials for the hypermultiplets can be generated via the Scherk-Schwarz mechanism and Fayet-Iliopoulos term in analogy to the undeformed case.

1 Introduction

Some backgrounds in string theory are known to trigger various types of space-time non-commutativity in the low-energy limit. For instance, a constant Neveu-Schwarz B-field background in type IIB string theory implies that the relevant low-energy dynamics is described by a gauge theory defined on non-commutative flat space, with $[x^i, x^j] = \mathrm{i}\theta^{ij}$, where θ^{ij} is a constant skew-symmetric matrix [1] (see also review [2]). Recently it was discovered that certain string backgrounds give rise to supersymmetric field theories living on superspaces with non-anticommuting Grassmann coordinates [3]–[6]. In particular, a specific four-dimensional compactification of the type IIB string in the presence of a constant self-dual graviphoton background $F^{\alpha\beta}$ yields a superspace whose odd coordinates obey the Clifford algebra $\{\theta^\alpha, \theta^\beta\} = \alpha'^2 F^{\alpha\beta}$ rather than the standard Grassmann algebra [5]. This superspace and supersymmetry realized in it must be of Euclidean signature since a real field strength can be self-dual ($F^{\alpha\beta}{\neq}0, F^{\dot\alpha\dot\beta}{=}0$) only in Euclidean (or Kleinian) but not in Minkowski space. The classical and quantum properties of theories defined on such nilpotently-deformed (or non-anticommutative) Euclidean $N{=}(\frac12,\frac12)$ superspace were analyzed in [5, 6] and in many subsequent works (see [7] for a review). As its most characteristic feature, this type of deformation breaks the original $N{=}(\frac12,\frac12)$ supersymmetry by half, i.e. down to $N{=}(\frac12,0)$, but preserves the important notion of chirality. The basic technical device of constructing the corresponding superfield theories is the Moyal-Weyl star product extended to Grassmann coordinates [8, 9, 10].

A natural step beyond the analysis of non-anticommutative $N{=}(\frac12,\frac12)$ superspace is the investigation of analogous nilpotent chiral deformations of Euclidean $N{=}(1,1)$ superfield theories. This study was initiated in [11, 12] and then continued and advanced in [13]–[16]. Both the D-type and Q-type deformations were considered, with either spinor covariant derivatives D^i_α or supersymmetry generators Q^i_α as the building blocks of the bi-differential Poisson operator defining the relevant star products [8, 9, 10].

The Q-deformations generically break the $N{=}(1,1)$ supersymmetry by half, i.e. down to $N{=}(1,0)$, but preserve both chirality and anti-chirality. The simplest $N{=}(1,1)$ Q-deformation is the singlet one ('QS-deformation'), based on the Poisson operator

$$P_s = -I \overleftarrow{Q}{}^i_\alpha \overrightarrow{Q}{}^\alpha_i \ , \qquad \text{with} \quad (P_s)^5 = 0 \ , \tag{1.1}$$

where I is a real parameter. While breaking half the supersymmetry, it preserves the internal $\mathrm{SU}(2)_R{\times}\mathrm{Spin}(4)$ symmetry, something impossible in the $N{=}(\frac12,\frac12)$ case. Furthermore, the QS-deformation as well can be given a stringy interpretation [13] along the line of [5, 3, 4, 17]. Namely, such a non-anticommutative $N{=}(1,1)$ superspace naturally arises for the $N{=}4$ superstring coupled to a complex axion background. The Q-deformations and their QS-subclass also preserve Grassmann harmonic analyticity [11, 12] which is the fundamental notion in theories with manifest extended supersymmetry [18, 19]. In view of these motivations, a complete understanding of the geometric and quantum structure of Q-deformed $N{=}(1,1)$ theories (and of their higher-N counterparts) is as important as the exploration of the analogous properties of non-anticommuting $N{=}(\frac12,\frac12)$ theories.

Until now, the main focus was on the Q-deformation of $N{=}(1,1)$ pure gauge theories. In [13], the detailed superfield and component structure of the QS-deformed $N{=}(1,1)$ U(1)

and U(n) gauge theories was explored.[1] In particular, an analog of the Seiberg-Witten (SW) map to quantities with undeformed gauge and supersymmetry transformation laws, both for the component fields and for the off-shell superfield strengths, was explicitly worked out.

An important (and not yet well-studied) class of $N=(1,1)$ theories are those including matter hypermultiplets interacting with themselves and with gauge multiplets. These theories are analogs of the coupled systems of chiral and gauge superfields of the $N=(\frac{1}{2},\frac{1}{2})$ case [5]. The first type of theories, i.e. self-interacting hypermultiplets, yields, in the bosonic sector, Euclidean versions of hyper-Kähler sigma models. The second type, i.e. hypermultiplets coupled to $N=(1,1)$ gauge multiplets, could be of relevance from the phenomenological point of view. The system of a gauge superfield minimally coupled to a hypermultiplet in the adjoint representation of the gauge group provides an off-shell $N=(1,1)$ superfield formulation of $N=(2,2)$ supersymmetric gauge theory which is the Euclidean analog of the renowned $N=4$ super Yang-Mills theory. With these motivations in mind, it is of obvious interest to elaborate on the structure of Q-deformed $N=(1,1)$ hypermultiplet theories. Note that, in contrast, the singlet D-deformation (which is the unique D-type deformation preserving Grassmann harmonic analyticity [11, 12]) does not at all affect $N=(1,1)$ analytic superfield Lagrangians, including those for hypermultiplets. So only the Q-deformations are of interest in the hypermultiplet context which is the subject of this paper.

In [11] we gave general recipes for constructing Q-deformations of superfield hypermultiplet actions but did not work out specific interesting examples of such models. In the present paper we consider, both in the superfield and component-field formulations, the QS-deformation of two simple actions with hypermultiplet and U(1) gauge superfields.[2]

Our first example, considered in Section 4, is the coupled system of an $N=(1,1)$ U(1) gauge superfield and a neutral hypermultiplet. Before turning on the QS-deformation, the action of this system is simply the sum of free actions for the gauge and hypermultiplet superfields, with one extra hidden on-shell $N=(1,1)$ supersymmetry extending the manifest $N=(1,1)$ supersymmetry to $N=(2,2)$. Once the deformation is turned on, the hypermultiplet starts to transform in a non-trivial way under the deformed U(1) gauge group: this transformation coincides with the 'non-abelian' part of the gauge superfield transformation. We pass to the physical component fields and find the corresponding SW-transformation. In terms of the undeformed fields the action radically simplifies. Yet, there still remains a new non-trivial interaction between the fermionic fields of the hypermultiplet and the gauge field (alongside with the known interaction between the gauge field and one of the scalar fields of the gauge multiplet [13, 15]). Besides the manifest unbroken $N=(1,0)$ supersymmetry, the resulting action proves to possess one more hidden $N=(1,0)$ supersymmetry with on-shell closure. Thus we are left with an unbroken $N=(2,0)$ supersymmetry in the QS-deformed $N=(2,2)$ U(1) gauge theory.[3]

[1] The QS-deformed U(1) theory was independently considered in [15].

[2] A toy model of QS-deformed self-interacting hypermultiplets, with the interaction vanishing in the undeformed limit, was recently considered in [20].

[3] Nilpotent Q-deformations of the on-shell superfield constraints of the Euclidean $N=(2,2)$ gauge theory were studied in [21].

Section 5 is devoted to the QS-deformation of a charged hypermultiplet in minimal interaction with a U(1) gauge multiplet. In the undeformed case, this interacting system possesses no extra supersymmetry besides the manifest $N{=}(1,1)$ one. The QS-deformation breaks the latter down to $N{=}(1,0)$. We analyze the component action of this deformed $N{=}(1,1)$ electrodynamics and the corresponding scalar field potential. The SW-transformation to undeformed fields exists in this case too.

An interesting notable feature of the two models considered is that there are only two inequivalent ways to implement the deformed U(1) gauge transformations on the hypermultiplet superfields. In Section 4 we deal with the 'adjoint' representation, while the deformed charged representations of Section 5 are equivalent to the 'fundamental' representation. One can treat a pair of mutually conjugate analytic hypermultiplet superfields as a doublet of an additional rigid 'Pauli-Gürsey' $\mathrm{SU}(2)_{PG}$ group [19]. The 'adjoint' U(1) transformation commutes with $\mathrm{SU}(2)_{PG}$, while the 'fundamental' U(1) transformation commutes only with $\mathrm{U}(1)_{PG} \subset \mathrm{SU}(2)_{PG}$ (just as in the undeformed case). Respectively, the first and the second model possess $\mathrm{SU}(2)_{PG}$ and $\mathrm{U}(1)_{PG}$ global symmetries in parallel with their $\mathrm{SU}(2)_R$ and Spin(4) automorphisms.

2 QS-deformation of N=(1,1) U(1) gauge theory

We start with a short overview of the Q-deformed Euclidean harmonic $N{=}(1,1)$ superspace and $N{=}(1,1)$ U(1) gauge theory following refs. [11] and [13].

The basic concepts of the $N{=}(1,1), D{=}4$ Euclidean harmonic superspace, which is an extension of the standard $N{=}(1,1)$ superspace by the $\mathrm{SU}(2)_R/\mathrm{U}(1)_R$ harmonics $u^\pm_i$, are collected in [11, 12, 13] (see also [19]). The standard (central) cordinates of the $N{=}(1,1)$ harmonic superspace are $(Z, u^\pm_i) = (x^m, \theta^\alpha_k, \bar\theta^{\dot\alpha k}, u^\pm_i)$. We shall also use the chiral coordinates

$$Z_L = (x^m_L, \theta^\alpha_k, \bar\theta^{\dot\alpha k}) , \quad x^m_L = x^m + \mathrm{i}(\sigma^m)_{\alpha\dot\alpha}\theta^\alpha_k\bar\theta^{\dot\alpha k} , \tag{2.1}$$

the chiral-analytic coordinates

$$Z_C = (x^m_L, \theta^{\pm\alpha} , \bar\theta^{\pm\dot\alpha}), \tag{2.2}$$

and the analytic coordinates

$$Z_A = (x^m_A, \theta^{\pm\alpha}, \bar\theta^{\pm\dot\alpha}) , \quad x^m_A \;=\; x^m_L - 2\mathrm{i}(\sigma^m)_{\alpha\dot\alpha}\theta^{-\alpha}\bar\theta^{+\dot\alpha} , \tag{2.3}$$

where $\theta^{\pm\alpha} = u^\pm_k\theta^{\alpha k}$ and $\bar\theta^{\pm\dot\alpha} = u^\pm_k\bar\theta^{\dot\alpha k}$. It should be stressed that all these coordinates are (pseudo)real with respect to the basic conjugation $\,\tilde{}\,$ [11]. For instance, the condition of reality can be consistently imposed on the Euclidean chiral superfields. An important invariant pseudoreal subspace of the harmonic superspace $(Z_A, u^\pm_i)$ is the analytic harmonic superspace

$$(\zeta, u^\pm_i) ,$$

where

$$\zeta = (x^m_A, \theta^{+\alpha}, \bar\theta^{+\dot\alpha}) .$$

The supersymmetry-preserving spinor derivatives $D^{\pm}_{\alpha}, \bar{D}^{\pm}_{\dot\alpha}$ and the harmonic derivatives $D^{\pm\pm}$ in different coordinate bases are given in [11, 13]. An analytic superfield satisfies the conditions

$$(D^+_\alpha, \bar{D}^+_{\dot\alpha})\Phi(Z, u^\pm_i) \;=\; 0 \quad \leftrightarrow \quad \Phi = \Phi_{\scriptscriptstyle A}(\zeta, u^\pm_i)\,. \tag{2.4}$$

In what follows it will be convenient to use harmonic projections of the supersymmetry generators

$$Q^k_\alpha = u^{+k}Q^-_\alpha - u^{-k}Q^+_\alpha, \quad \bar{Q}_{\dot\alpha k} = u^+_k\bar{Q}^- - u^-_k\bar{Q}^+. \tag{2.5}$$

For instance, in the analytic coordinates

$$Q^+_\alpha = \partial_{-\alpha} - 2\mathrm{i}\bar\theta^{+\dot\alpha}\partial_{\alpha\dot\alpha}, \quad Q^-_\alpha = -\partial_{+\alpha}\,, \tag{2.6}$$

where $\partial_{\pm\alpha} = \partial/\partial\theta^{\pm\alpha}$ and $\partial_{\alpha\dot\alpha} = (\sigma_m)_{\alpha\dot\alpha}\partial_m$.

In this paper we shall deal with the Q-singlet (or QS-) deformation which is associated with the SO(4)×SU(2) invariant Poisson bracket

$$AP_sB = -I(-1)^{p(A)}Q^k_\alpha AQ^\alpha_k B = -I(-1)^{p(A)}\,(Q^+_\alpha AQ^{-\alpha}B + Q^{-\alpha}AQ^+_\alpha B). \tag{2.7}$$

The noncommutative QS-star product of two analytic superfields has the following simple form:

$$\Lambda \star \Phi = \Lambda\Phi + \Lambda P_s\Phi + \tfrac{1}{2}\Lambda P^2_s\Phi\,. \tag{2.8}$$

Since $\partial^\alpha_- A = \partial^\alpha_- B = 0$ in the analytic basis, we can omit $\partial_{-\alpha}$ in Q^+_α in this basis. In particular,

$$AP_sB = 2\mathrm{i}I(-1)^{p(A)}\bar\theta^{+\dot\alpha}(\partial^\alpha_+ A\partial_{\alpha\dot\alpha}B - \partial_{\alpha\dot\alpha}A\partial^\alpha_+ B)\,. \tag{2.9}$$

In the $\star$-commutator of two bosonic analytic superfields $[A, B]_\star$ with $[A, B] = 0$ only the term $\sim P_s$ survives:

$$[A, B]_\star = 2AP_sB\,, \tag{2.10}$$

while the $\star$-anticommutator reads

$$\{A, B\}_\star = 2AB + AP^2_sB. \tag{2.11}$$

We shall make use of these general formulas in Sections 4 and 5.

The basic superfield of the $N{=}(1,1)$ gauge theory is the analytic gauge potential V^{++}. The QS-deformed gauge transformation of the U(1) potential V^{++} in the analytic basis reads

$$\begin{aligned}\delta_\Lambda V^{++} \;&=\; D^{++}\Lambda + [V^{++}, \Lambda]_\star = D^{++}\Lambda + 2V^{++}P_s\Lambda \\ &=\; D^{++}\Lambda + 4\mathrm{i}I\bar\theta^{+\dot\alpha}(\partial^\alpha_+ V^{++}\partial_{\alpha\dot\alpha}\Lambda - \partial_{\alpha\dot\alpha}V^{++}\partial^\alpha_+\Lambda)\,.\end{aligned} \tag{2.12}$$

Here Λ is an analytic gauge parameter and $\tilde\Lambda = -\Lambda$. The gauge potential in the Wess-Zumino gauge has the following $\theta^{+\alpha}, \bar\theta^{+\dot\alpha}$-expansion

$$\begin{aligned}V^{++}_{\scriptscriptstyle WZ} &= (\theta^+)^2\bar\phi + (\bar\theta^+)^2\phi + 2\theta^+\sigma_m\bar\theta^+A_m + 4(\theta^+)^2\bar\theta^+_{\dot\alpha}u^-_k\bar\Psi^{\dot\alpha k} \\ &+ 4(\bar\theta^+)^2\theta^{+\alpha}u^-_k\Psi^k_\alpha + 3(\theta^+)^2(\bar\theta^+)^2u^-_ku^-_l\mathcal{D}^{kl}\,,\end{aligned} \tag{2.13}$$

with all the component fields being functions of x^m_A. The residual U(1) gauge transformations with the parameter $\Lambda_r = ia(x_A)$ act on the component fields in (2.13) as

$$\begin{aligned}
&\delta_r\bar\phi = 0, \quad \delta_r\phi = -8IA_m\partial_m\, a\,, \quad \delta_r A_m = (1+4I\bar\phi)\partial_m\, a\,,\\
&\delta_r\Psi^k_\alpha = -4I\bar\Psi^{\dot\alpha k}\partial_{\alpha\dot\alpha}a \quad \delta_r\bar\Psi^k_{\dot\alpha} = 0\,, \quad \delta_r\mathcal{D}^{kl} = 0\,.
\end{aligned}\tag{2.14}$$

The deformed $N=(1,0)$ supersymmetry transformations of the U(1) gauge multiplet component fields defined in (2.13) are as follows [13]:

$$\begin{aligned}
&\delta_\epsilon\phi = 2\epsilon^{\alpha k}\Psi_{\alpha k}\,, \quad \delta_\epsilon\bar\phi = 0\,, \quad \delta_\epsilon A_m = \epsilon^{\alpha k}(\sigma_m)_{\alpha\dot\alpha}\bar\Psi^{\dot\alpha}_k\,,\\
&\delta_\epsilon\Psi^k_\alpha = -\epsilon_{\alpha l}\mathcal{D}^{kl} + \tfrac{1}{2}(1+4I\bar\phi)(\sigma_{mn}\epsilon^k)_\alpha F_{mn} - 4\mathrm{i}I\epsilon^k_\alpha A_m\partial_m\bar\phi\,,\\
&\delta_\epsilon\bar\Psi^k_{\dot\alpha} = -\mathrm{i}\epsilon^{\alpha k}(1+4I\bar\phi)\partial_{\alpha\dot\alpha}\bar\phi\,,\\
&\delta_\epsilon\mathcal{D}^{kl} = \mathrm{i}\partial_m[(\epsilon^k\sigma_m\bar\Psi^l + \epsilon^l\sigma_m\bar\Psi^k)(1+4I\bar\phi)]\,,
\end{aligned}\tag{2.15}$$

where $F_{mn} = \partial_m A_n - \partial_n A_m$. They are produced by the transformation of V^{++}_{WZ} which is a sum of the standard supertranslation piece and the compensating gauge transformation with the parameter

$$\Lambda_\epsilon = 2(\epsilon^-\theta^+)\bar\phi - 2\bar\theta^+_{\dot\alpha}\epsilon^-_\alpha(A^{\alpha\dot\alpha} + 2\theta^{+\alpha}\bar\Psi^{-\dot\alpha}) + 2(\bar\theta^+)^2[(\epsilon^-\Psi^-) + (\epsilon^-\theta^+)\mathcal{D}^{--}]\,. \tag{2.16}$$

The Seiberg-Witten (SW) transformation to the undeformed U(1) gauge supermultiplet $\varphi, \bar\phi, a_m, \psi^\alpha_k, \bar\psi^{\dot\alpha}_k, d^{kl}$ with the standard gauge and N=(1,0) supersymmetry transformation properties (corresponding to the choice I=0 in (2.14), (2.15)) is defined as the following set of relations

$$\begin{aligned}
&\phi = (1+4I\bar\phi)^2\varphi - 4I(1+4I\bar\phi)^{-1}[A^2_m + 4I^2(\partial_m\bar\phi)^2]\,,\\
&A_m = (1+4I\bar\phi)a_m\,, \quad \bar\Psi^k_{\dot\alpha} = (1+4I\bar\phi)\bar\psi^k_{\dot\alpha}\,,\\
&\Psi^k_\alpha = (1+4I\bar\phi)^2\psi^k_\alpha - 4I(1+4I\bar\phi)a_{\alpha\dot\alpha}\bar\psi^{\dot\alpha k}\,,\\
&\mathcal{D}^{kl} = (1+4I\bar\phi)^2 d^{kl} - 8I(1+4I\bar\phi)\bar\psi^k_{\dot\alpha}\bar\psi^{\dot\alpha l}\,.
\end{aligned}\tag{2.17}$$

Using them, one can express the component Lagrangian of the deformed U(1) gauge theory through the standard undeformed free N=(1,1) gauge theory Lagrangian [13, 15]

$$L_g = (1+4I\bar\phi)^2[-\tfrac{1}{2}\varphi\,\Box\bar\phi + \tfrac{1}{4}f^2_{mn} + \tfrac{1}{8}\varepsilon_{mnrs}f_{mn}f_{rs} - \mathrm{i}\psi^\alpha_k\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha k} + \tfrac{1}{4}(d^{kl})^2]\,, \tag{2.18}$$

where $f_{mn} = \partial_m a_n - \partial_n a_m$. After redefining the involved fields as

$$\hat\varphi = (1+4I\bar\phi)^2\varphi\,, \quad \hat\psi^{\,\alpha}_{\,k} = (1+4I\bar\phi)^2\psi^\alpha_k\,, \quad \hat d^{\,kl} = (1+4I\bar\phi)d^{kl}\,, \tag{2.19}$$

the Lagrangian (2.18) acquires the form in which it differs from the free Lagrangian only by a simple interaction term:

$$L_g = -\tfrac{1}{2}\hat\varphi\,\Box\bar\phi - \mathrm{i}\hat\psi^{\,\alpha}_{\,k}\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha k} + \tfrac{1}{4}(1+4I\bar\phi)^2\left(f^2_{mn} + \tfrac{1}{2}\varepsilon_{mnrs}f_{mn}f_{rs}\right) + \tfrac{1}{4}(\hat d^{\,kl})^2\,. \tag{2.20}$$

Note that the fields $\bar\phi, \hat\psi^{\,\alpha}_{\,k}, \bar\psi^{\dot\alpha}_k$ and $\hat d^{\,kl}$ satisfy the free equations.

3 Representations of the QS-deformed gauge group

The free q^+ hypermultiplet action of the ordinary harmonic theory [19] is not deformed in the non-anticommutative superspace [11, 12]:

$$S_0(q^+) \;=\; -\int \mathrm{d}u\,\mathrm{d}\zeta^{-4}\;\tilde{q}^{\,+}D^{++}q^+ = \tfrac{1}{2}\int \mathrm{d}u\,\mathrm{d}\zeta^{-4}\;q^+_a D^{++}q^{+a}\,. \tag{3.1}$$

Here $\mathrm{d}\zeta^{-4} = \mathrm{d}^4x_{A}(D^-)^4$ and the additional $\mathrm{SU}(2)_{PG}$ indices $a, b = 1, 2$ are introduced, $q^{+a} = \varepsilon^{ab}q^+_b = (\tilde{q}^+, q^+) = \widetilde{q^+_a}$. After passing to the component fields, integrating over $\theta^+_\alpha, \bar{\theta}^+_{\dot\alpha}$ and eliminating the infinite tower of the auxiliary fields by their algebraic equations of motion, the superfield free action (3.1) yields the free action for (4+8) physical fields of the hypermultiplet, viz. scalars $f^{ak}(x_{A})$ and fermions $\rho^a_\alpha(x_{A})$ and $\chi^a_{\dot\alpha}(x_{A})$. We do not quote it here; it can be easily reproduced as the $I \to 0$ limit of the QS-deformed actions to be given below.

Let us discuss how the QS-deformed U(1) gauge transformations can be implemented on the superfield q^{+a}. Obviously, these transformations should have the same Lie bracket structure as the the QS-deformed U(1) gauge transformations (2.12) of V^{++}. It is easy to see that the Lie bracket of two such transformations has again the form (2.12) with $\Lambda_{br} = [\Lambda_2, \Lambda_1]_\star$. Then a simple analysis shows that only two non-equivalent realizations of the deformed U(1) gauge group with such Lie bracket structure are possible for the hypermultiplet doublet q^{+a} (besides the trivial realization $\delta_\Lambda q^{+a} = 0$)

$$1.\quad \delta_\Lambda q^{+a} = [q^{+a}, \Lambda]_\star\,, \tag{3.2}$$

$$2.\quad \delta_\Lambda q^{+a} = \tfrac{1}{2}[q^{+a}, \Lambda]_\star + \tfrac{1}{2}(\tau_3)^a_b\{q^{+b}, \Lambda\}_\star\,, \tag{3.3}$$

where τ_3 is the diagonal Pauli matrix. These representations of the deformed U(1) group can be naturally called ‘adjoint’ and ‘fundamental’, respectively. Indeed, (3.2) has the same form as the non-abelian part of the deformed U(1) transformation (2.12) of the gauge superfield, while (3.3) can be equivalently rewritten as

$$\delta_\Lambda q^+ = -\Lambda \star q^+\,, \quad \delta_\Lambda \tilde{q}^+ = \tilde{q}^+ \star \Lambda\,. \tag{3.4}$$

An interesting peculiarity of the QS-deformed realization (3.3), (3.4) of the U(1) gauge group is that the U(1) charge of q^+ and $\tilde{q}^+$ in it is fixed up to a finite $\mathrm{SU}(2)_{PG}$ rotation of this pair. In the commutative limit we are left with the U(1) charges ∓ 1 for q^+ and $\tilde{q}^+$. On the contrary, in the undeformed case one can ascribe to q^+ an arbitrary real charge e corresponding to the U(1) gauge transformation $\delta_\Lambda q^+ = e\,\Lambda q^+\,,\ \delta_\Lambda \tilde{q}^+ = -e\,\Lambda \tilde{q}^+\,.$ This difference is related to the fact that QS-deformed U(1) transformations have a non-trivial closure on off-shell superfields (before passing to the WZ gauge [4]) and this closure should be the same for all involved superfields, i.e. for V^{++} and q^{+a}. We also note that (3.3) is a particular case of the more general two-parameter family of realizations with the same

[4] The residual U(1) gauge group of the WZ gauge is abelian despite the fact that the full QS-deformed superfield U(1) group has a non-trivial Lie bracket structure.

Lie bracket structure

$$\delta_\Lambda q^+ = -\tfrac{1}{2}(\cos\mu + 1)\Lambda \star q^+ - \tfrac{1}{2}e^{i\alpha}\sin\mu\,\Lambda \star \tilde{q}^+$$
$$+\tfrac{1}{2}(1-\cos\mu) q^+ \star \Lambda - \tfrac{1}{2}e^{i\alpha}\sin\mu\,\tilde{q}^+ \star \Lambda \qquad \text{(and c.c.)}\,, \tag{3.5}$$

where α and μ are arbitrary real parameters. Eqs. (3.3), (3.4) correspond to the special choice $\mu = 0$. However, it is easy to see that (3.5) can be generated from (3.3), (3.4) by a finite $\mathrm{SU}(2)_{PG}/\mathrm{U}(1)_{PG}$ rotation of the pair $(\tilde{q}^+, q^+)$. So all such realizations are equivalent and, without loss of generality, we can choose (3.3), (3.4) to deal with. The 'adjoint' realization (3.2) clearly commutes with the whole $\mathrm{SU}(2)_{PG}$, while (3.3) ((3.4)) commutes only with $\mathrm{U}(1)_{PG} \subset \mathrm{SU}(2)_{PG}$. So the hypermultiplet theories associated with these two different realizations of the QS-deformed U(1) gauge group respect $\mathrm{SU}(2)_{PG}$ and $\mathrm{U}(1)_{PG}$ as global internal symmetries. In fact, the second realization can be regarded just as the QS-deformed gauging of this latter U(1).

One can wonder whether the charged hypermultiplets with an arbitrary value of the charge e can be properly QS-deformed. The answer is affirmative, but somewhat surprising. For this one should modify the QS-deformed U(1) gauge transformation law (2.12) in the following way

$$\delta_\Lambda V^{++} \;=\; D^{++}\Lambda + e\,[V^{++}, \Lambda]_\star \tag{3.6}$$

and the hypermultiplet transformation laws (3.2) and (3.3) as

$$1.\quad \delta_\Lambda q^{+a} = e\,[q^{+a}, \Lambda]_\star\,, \tag{3.7}$$
$$2.\quad \delta_\Lambda q^{+a} = \frac{1}{2}\,e\,[q^{+a}, \Lambda]_\star + \frac{1}{2}\,e\,(\tau_3)^a_b\{q^{+b}, \Lambda\}_\star\,. \tag{3.8}$$

Eq. (3.8) can be rewritten as

$$\delta_\Lambda q^+ = -e\,\Lambda \star q^+\,, \quad \delta_\Lambda \tilde{q}^+ = e\,\tilde{q}^+ \star \Lambda \tag{3.9}$$

and it goes into the standard U(1) transformation law of the hypermultiplet of the charge $|e|$ in the commutative limit. Any gauge transformation from the one-parameter family (3.6) in this limit goes into the standard abelian transformation of V^{++}. The modified transformation laws have the same Lie bracket structure for all superfields, now with the bracket parameter $e[\Lambda_2, \Lambda_1]_\star$. For the deformed U(1) gauge system, as well as in the coupled system of the U(1) gauge superfield and 'adjoint' hypermultiplet, this modification is in fact unessential: the parameter e can be removed from the superfield transformation laws and the relevant invariant actions by rescaling the deformation parameter as $eI \to I'$. In the system of V^{++} and the 'fundamental' hypermultiplet the charge e cannot be removed since it is present there already in the undeformed limit. Thus e can be treated as an additional deformation parameter of the analytical superfield gauge group, and gauge transformations for different e are not equivalent. Note that both in the undeformed and deformed cases one can introduce the gauge U(1) group in such a way that it will commute with full $\mathrm{SU}(2)_{PG}$. This can be achieved by introducing *two* independent hypermultiplets, which amounts to making q^{+a} complex, $\widetilde{q^+_a} \neq q^{+a}$. Then one can gauge U(1) which

multiplies the whole q^{+a} by phase and so commutes with $\mathrm{SU}(2)_{PG}$ acting on the doublet indices. The corresponding analog of the QS-deformed transformation rules (3.9) is

$$\delta_\Lambda q^{+a} = -e\,\Lambda \star q^{+a}\,, \quad \delta_\Lambda \tilde{q}^+_a = e\,\tilde{q}^+_a \star \Lambda\,. \tag{3.10}$$

In this paper we concentrate mainly on the simplest case of one hypermultiplet.

The QS-deformed superfield hypermultiplet actions corresponding to the two realizations of U(1) gauge group presented above (for arbitrary e) are constructed according to the general rule of ref. [11], viz. via the replacement

$$D^{++}q^{+\,a} \;\Rightarrow\; \nabla^{++}q^{+\,a} \tag{3.11}$$

in the action (3.1), where, for two options (3.7), (3.8),

$$1. \quad \nabla^{++}q^{+\,a} = D^{++}q^{+\,a} + e\,[V^{++}, q^{+a}]_\star = D^{++}q^{+\,a} + 2e\,V^{++}P_s q^{+\,a}\,, \tag{3.12}$$

$$2. \quad \nabla^{++}q^{+\,a} = D^{++}q^{+\,a} + \frac{e}{2}\,[V^{++}, q^{+a}]_\star - \frac{e}{2}\,(\tau_3)^a_b\{V^{++}, q^{+b}\}_\star\,. \tag{3.13}$$

The full QS-deformed action in both cases is the sum of the superfield action of the QS-deformed U(1) gauge multiplet given in [13, 15] and the gauge invariant hypermultiplet action

$$S(V,q) = \tfrac{1}{2}\int \mathrm{d}u\,\mathrm{d}\zeta^{-4}\; q^+_a\,\nabla^{++}q^{+a}\,. \tag{3.14}$$

Further we shall consider the component structure of both types of the gauge multiplet-hypermultiplet action.

4 Deformed neutral hypermultiplet

In the explicit form, the invariant action (3.14) for this case reads

$$S_n(V,q) = \tfrac{1}{2}\int \mathrm{d}u\mathrm{d}\zeta^{-4} q^+_a\;\left[D^{++}q^{+a} + 4\mathrm{i}I\,\bar\theta^{+\dot\alpha}(\partial^\alpha_+ V^{++}\partial_{\alpha\dot\alpha}q^{+a} - \partial_{\alpha\dot\alpha}V^{++}\partial^\alpha_+ q^{+a})\right]\,, \tag{4.1}$$

where we made use of eqs. (2.10), (2.9) and (3.12) and put $e=1$ since the dependence on e in the considered case can be absorbed into a redefinition of I. To obtain the component action, we should substitute into (4.1) the WZ form of V^{++}, eq. (2.13), and the θ-expansion of q^{+a}

$$\begin{aligned} q^{+a} &= f^{+a} + \theta^{+\alpha}\pi^a_\alpha + \bar\theta^+_{\dot\alpha}\kappa^{\dot\alpha a} + \theta^+\sigma_m\bar\theta^+ r^{-a}_m + (\theta^+)^2 g^{-a} + (\bar\theta^+)^2 h^{-a} \\ &+ (\bar\theta^+)^2\theta^{+\alpha}\Sigma^{--a}_\alpha + (\theta^+)^2\bar\theta^{+\dot\alpha}\bar\Sigma^{--a}_{\dot\alpha} + (\theta^+)^2(\bar\theta^+)^2\omega^{-3\,a}\,, \end{aligned} \tag{4.2}$$

where all component fields are functions of x_A and u. Then we should integrate in (4.1) over $\theta^+, \bar\theta^+$, and eliminate the infinite number of the auxiliary fields contained in (4.2) using the appropriate non-dynamical equations of motion. It is convenient to extract the latter directly from the superfield equation of motion

$$D^{++}q^{+a} + 4\mathrm{i}I\bar\theta^{+\dot\alpha}(\partial^\alpha_+ V^{++}\partial_{\alpha\dot\alpha}q^{+a} - \partial_{\alpha\dot\alpha}V^{++}\partial^\alpha_+ q^{+a}) = 0\,. \tag{4.3}$$

Skipping details, we find the following solution for the components in (4.2) in terms of the remaining 4 physical bosonic fields $f^{ia}(x_{\scriptscriptstyle A})$ and 8 physical fermions $\rho^a_\alpha(x_{\scriptscriptstyle A})$, $\chi^{\dot\alpha a}(x_{\scriptscriptstyle A})$

$$
\begin{aligned}
& f^{+a} = f^{ak}u^+_k\,, \quad \pi^a_\alpha = \rho^a_\alpha\,, \quad \kappa^{\dot\alpha a} = \chi^{\dot\alpha a}\,, \quad r^{-a}_m = r^{ak}_m u^-_k\,, \quad g^{-a} = 0\,, \\
& h^{-a} = h^{ak}u^-_k\,, \quad \Sigma^{--a}_\alpha = \Sigma^{kl\,a}_\alpha u^-_k u^-_l\,, \quad \bar\Sigma^{--a}_{\dot\alpha} = 0\,, \quad \omega^{-3\,a} = 0\,, \\
& r^{ak}_m = 2\mathrm{i}(1+4I\bar\phi)\partial_m f^{ak}\,, \quad h^{ak} = -8\mathrm{i}\,I A_m\partial_m f^{ak}\,, \\
& \Sigma^{kl\,a}_\alpha = -4\mathrm{i}\,I(\bar\Psi^{\dot\alpha k}\partial_{\alpha\dot\alpha}f^{al} + \bar\Psi^{\dot\alpha l}\partial_{\alpha\dot\alpha}f^{ak})\,.
\end{aligned} \tag{4.4}
$$

Actually, for deducing the component results below it is of no need to know the expilcit form of the solution for h^{-a} and Σ^{--a}; we have presented it for completeness.

After substituting (4.4) into the original action and performing there integration over harmonics, the final Lagrangian in the x-space is written in terms of the physical fields as

$$
\begin{aligned}
L_1 = {}& \tfrac{1}{2}(1+4I\bar\phi)^2\,\partial_m f^{ak}\partial_m f_{ak} + \tfrac{1}{2}\mathrm{i}(1+4I\bar\phi)\rho^{\alpha a}\partial_{\alpha\dot\alpha}\chi^{\dot\alpha}_a + 4\mathrm{i}I\bar\Psi^{\dot\alpha}_k\rho^\alpha_a\partial_{\alpha\dot\alpha}f^{ak} \\
& + 2\mathrm{i}I\rho^{\alpha a}A_m\partial_m\rho_{\alpha a} + \mathrm{i}I\rho^{\beta a}\rho^\alpha_a\partial_{(\alpha\dot\alpha}A^{\dot\alpha}_{\beta)}\,.
\end{aligned} \tag{4.5}
$$

Note that only the components $\bar\phi, A_m$ and $\bar\Psi^{\dot\alpha}_k$ of the gauge multiplet interact with the hypermultiplet fields in this model. We shall see soon that in the sum of L_1 and the U(1) gauge multiplet Lagrangian L_g, eq.(2.18), most of the interaction terms can be removed by means of the proper redefinition of fields.

Let us discuss symmetries of (4.5). The adjoint hypermultiplet gauge transformation (3.2) in the unfolded form reads

$$
\delta_\Lambda q^{+a} = 4\mathrm{i}I\bar\theta^+_{\dot\alpha}(\partial^{\alpha\dot\alpha}\Lambda\partial_{+\alpha}q^{+a} - \partial_{+\alpha}\Lambda\partial^{\alpha\dot\alpha}q^{+a})\,, \tag{4.6}
$$

while the unbroken $N{=}(1,0)$ supersymmetry transformation is simply

$$
\delta_\epsilon q^{+\,a} = \left(\epsilon^{-\alpha}Q^+_\alpha - \epsilon^{+\alpha}Q^-_\alpha\right)q^{+\,a} = \left(\epsilon^{+\alpha}\partial_{+\alpha} - 2i\epsilon^{-\alpha}\bar\theta^{+\dot\alpha}\partial_{\alpha\dot\alpha}\right)q^{+\,a}\,, \tag{4.7}
$$

where $\epsilon^{\pm\alpha} = \epsilon^{ai}u^\pm_i$. In the WZ-gauge for the U(1) potential (2.13) the corresponding residual gauge and $N{=}(1,0)$ supersymmetry transformations of the hypermultiplet are

$$
\delta_r q^{+a} = -4I\bar\theta^+_{\dot\alpha}\partial^{\alpha\dot\alpha}a(x_{\scriptscriptstyle A})\partial_{+\alpha}q^{+a}, \tag{4.8}
$$

$$
\delta_\epsilon q^{+a} = (\epsilon^{+\alpha}\partial_{+\alpha} + 2i\bar\theta^+_{\dot\alpha}\epsilon^-_\alpha\partial^{\alpha\dot\alpha})q^{+a} + 4\mathrm{i}I\bar\theta^+_{\dot\alpha}(\partial_{+\alpha}\Lambda_\epsilon\partial^{\alpha\dot\alpha}q^{+a} - \partial^{\alpha\dot\alpha}\Lambda_\epsilon\partial_{+\alpha}q^{+a})\,, \tag{4.9}
$$

where the field-dependent compensating gauge parameter Λ_ϵ was defined in (2.16). After making use of eqs. (4.4) for the component fields in these formulas we obtain the residual gauge transformations of the physical fields of q^{+a} as

$$
\delta_r f^{ak} = 0, \quad \delta_r\rho^a_\alpha = 0, \quad \delta_r\chi^{\dot\alpha a} = -4I\partial^{\alpha\dot\alpha}a\,\rho^a_\alpha \tag{4.10}
$$

and the corresponding $N = (1,0)$ transformations as

$$
\delta_\epsilon f^{ak} = \epsilon^{\alpha k}\rho^a_\alpha, \quad \delta_\epsilon\rho^a_\alpha = 0, \quad \delta_\epsilon\chi^a_{\dot\alpha} = 2\mathrm{i}\epsilon^{\alpha k}(1+4I\bar\phi)\partial_{\alpha\dot\alpha}f^a_k\,. \tag{4.11}
$$

It is straightforward to construct an analog of the SW-type transform for the physical fields of the deformed hypermultiplet

$$
\begin{aligned}
& f_0^{ak}=(1+4I\bar{\phi})f^{ak}, \qquad \rho_0^{\alpha a}=(1+4I\bar{\phi})\rho^{\alpha a}, \\
& \chi_0^{\dot{\alpha}a}=\chi^{\dot{\alpha}a}+4I(1+4I\bar{\phi})^{-1}A^{\alpha\dot{\alpha}}\rho_\alpha^a-8I(1+4I\bar{\phi})^{-1}\bar{\Psi}^{\dot{\alpha}k}f_k^a\,. \qquad (4.12)
\end{aligned}
$$

The redefined hypermultiplet fields $f_0^{ak}, \rho_0^{\alpha a}, \chi_0^{\dot{\alpha}a}$ possess the standard undeformed transformation properties: they are neutral with respect to the gauge group U(1), i.e. their $a(x)$ variations are equal to zero, and their $N=(1,0)$ SUSY transformations look as the $I=0$ case of (4.11).

Let us pass to the 'undeformed' component fields in the Lagrangian (4.5). The straightforward computation shows that, up to a total derivative, it acquires the following simple form

$$
\begin{aligned}
L_1 = & \tfrac{1}{2}\,\partial_m f_0^{ak}\partial_m f_{0\,ak}+\tfrac{1}{2}\mathrm{i}\rho_0^{\alpha a}\partial_{\alpha\dot{\alpha}}\chi_{0\,a}^{\dot{\alpha}}+2\mathrm{i}I(1+4I\bar{\phi})^{-1}\rho_0^{\beta a}\rho_{0\,a}^{\alpha}\partial_{(\alpha\dot{\alpha}}a_{\beta)}^{\dot{\alpha}} \\
& +2I(1+4I\bar{\phi})^{-1}(f_0^{ak}f_{0\,ak})\Box\bar{\phi}+4\mathrm{i}I(1+4I\bar{\phi})^{-1}(\rho_0^{\alpha a}f_{0\,ak})\partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}k}\,. \qquad (4.13)
\end{aligned}
$$

Now it is easy to observe that in the total gauge multiplet-hypermultiplet Lagrangian $L=L_g+L_1$ the last two terms in (4.13) can be removed by the appropriate redefinition of the fields φ and ψ_k^α of the U(1) gauge multiplet

$$
\begin{aligned}
& \hat{\varphi}=(1+4I\bar{\phi})^2\varphi-4I(1+4I\bar{\phi})^{-1}(f_0^{ak}f_{0\,ak})\,, \\
& \hat{\psi}_k^\alpha=(1+4I\bar{\phi})^2\psi_k^\alpha-4I(1+4I\bar{\phi})^{-1}(\rho_0^{\alpha a}f_{0\,ak})\,. \qquad (4.14)
\end{aligned}
$$

In terms of new fields the total on-shell Lagrangian can be rewritten as the sum of the free gauge multiplet-hypermultiplet action and the simple interaction term

$$
L=L_g+L_1=L_0+L_{int}\,, \qquad (4.15)
$$

$$
\begin{aligned}
L_0 = & -\tfrac{1}{2}\hat{\varphi}\Box\bar{\phi}+\tfrac{1}{2}\,\partial_m f_0^{ak}\partial_m f_{0\,ak}-\tfrac{1}{16}f^{\alpha\beta}f_{\alpha\beta}-\mathrm{i}\hat{\psi}_k^\alpha\partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}k} \qquad (4.16)\\
& +\tfrac{1}{2}\mathrm{i}\rho_0^{\alpha a}\partial_{\alpha\dot{\alpha}}\chi_{0\,a}^{\dot{\alpha}}+\tfrac{1}{4}(\hat{d}_{kl})^2\,,
\end{aligned}
$$

$$
L_{int}=-\tfrac{1}{2}\,I\,\bar{\phi}(1+2I\bar{\phi})\,f^{\alpha\beta}f_{\alpha\beta}+I\,(1+4I\bar{\phi})^{-1}\,\rho_0^{\beta a}\rho_{0\,a}^{\alpha}f_{\alpha\beta}\,, \qquad (4.17)
$$

where $f_{\alpha\beta}=\mathrm{i}\partial_{\alpha\dot{\alpha}}a_\beta^{\dot{\alpha}}+\mathrm{i}\partial_{\beta\dot{\alpha}}a_\alpha^{\dot{\alpha}}=(\sigma_{mn})_{\alpha\beta}f_{mn}$ and $\hat{d}_{kl}=(1+4I\bar{\phi})d_{kl}$. Note that the corresponding equations for the fields $\bar{\phi}, f_0^{ak}, \bar{\psi}^{\dot{\alpha}k}, \rho_0^{\alpha a}$ and $\hat{d}_{kl}$ are free.

In the limit $I=0$ the Lagrangian (4.15) is reduced to a sum of free Lagrangians of the vector gauge multiplet and hypermultiplet, and so it represents the Euclidean $N{=}(2,2)$ supersymmetric abelian gauge theory. Hence in this limit it should exhibit a hidden on-shell $N{=}(1,1)$ supersymmetry which forms $N{=}(2,2)$ supersymmetry together with the manifest $N{=}(1,1)$. At $I\neq 0$ the Lagrangian (4.15) can be treated as a QS-deformed version of the $N{=}(2,2)$ gauge theory Lagrangian, and it is expected to respect half of the original $N{=}(2,2)$ supersymmetry. Indeed, it can be checked that (4.15) is invariant, up to a total derivative, under the following extra $N{=}(1,0)$ supersymmetry:

$$
\begin{aligned}
& \delta_\eta\hat{\varphi}=0\,, \quad \delta_\eta\bar{\phi}=-2\eta^{\alpha a}\rho_{0\alpha a}\,, \quad \delta_\eta a_{\alpha\dot{\alpha}}=2\eta_\alpha^a\chi_{0a\dot{\alpha}}\,, \quad \delta_\eta\hat{\psi}_k^\alpha=0\,, \quad \delta_\eta\hat{d}_{kl}=0\,, \\
& \delta_\eta\bar{\psi}^{\dot{\alpha}k}=2\mathrm{i}\eta_\alpha^a\partial^{\alpha\dot{\alpha}}f_{0a}^k\,, \quad \delta_\eta f_0^{ak}=-2\eta^{\alpha a}\hat{\psi}_\alpha^k\,, \quad \delta_\eta\chi_{0a}^{\dot{\alpha}}=-2\mathrm{i}\eta_{\alpha a}\partial^{\alpha\dot{\alpha}}\hat{\varphi}\,, \\
& \delta_\eta\rho_0^{\alpha a}=(1+4I\bar{\phi})^2\eta_\beta^a f^{\alpha\beta}-8I(1+4I\bar{\phi})^{-1}\eta_\beta^a\rho_0^{\beta b}\rho_{0b}^\alpha\,, \qquad (4.18)
\end{aligned}
$$

where $\eta^{\alpha a}$ are the corresponding Grassmann parameters. In terms of the redefined fields the manifest $N{=}(1,0)$ supersymmetry is realized by the transformations

$$\begin{aligned}
&\delta_\epsilon\hat\varphi = 2\epsilon^{\alpha k}\hat\psi_{\alpha k}, \quad \delta\bar\phi = 0\,, \quad \delta_\epsilon a_{\alpha\dot\alpha} = 2\epsilon^k_\alpha\bar\psi_{\dot\alpha k}\,,\\
&\delta_\epsilon\hat\psi^\alpha_k = (1+4I\bar\phi)\epsilon^{\alpha l}\hat d_{kl} + \tfrac12(1+4I\bar\phi)^2\epsilon_{\beta k}f^{\alpha\beta} - 4I(1+4I\bar\phi)^{-1}\epsilon_{\beta k}(\rho_0^{\alpha a}\rho^\beta_{0a})\,,\\
&\delta_\epsilon\bar\psi^{\dot\alpha k} = \mathrm{i}\epsilon^k_\alpha\partial^{\alpha\dot\alpha}\bar\phi\,, \quad \delta_\epsilon\hat d_{kl} = \mathrm{i}(1+4I\bar\phi)(\epsilon^\alpha_k\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha}_l + \epsilon^\alpha_l\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha}_k)\,,\\
&\delta_\epsilon f^{ak}_0 = \epsilon^{\alpha k}\rho^a_{0\alpha}\,, \quad \delta_\epsilon\chi^{\dot\alpha}_{0a} = -2\mathrm{i}\epsilon^k_\alpha\partial^{\alpha\dot\alpha}f_{0ak}\,, \quad \delta_\epsilon\rho^{\alpha a}_0 = 0\,.
\end{aligned} \tag{4.19}$$

The additional η-transformations commute on-shell with themselves and also with the $N{=}(1,0)$ ϵ-transformations, for instance,

$$\begin{aligned}
&(\delta_\epsilon\delta_\eta - \delta_\eta\delta_\epsilon)f^{ak}_0 = 2\eta^{\alpha a}(1+4I\bar\phi)\epsilon_{\alpha l}\hat d^{kl} = 0\,,\ (\delta_\eta\delta_\epsilon - \delta_\epsilon\delta_\eta)\hat\psi^\alpha_k = -8I\eta^{\beta a}\rho_{0\beta a}\epsilon^{\alpha l}\hat d_{kl}\\
&- \mathrm{i}(1+4I\bar\phi)^2\epsilon_{\beta k}[\eta^{\beta a}(\partial^{\alpha\dot\alpha}\chi_{0\dot\alpha a} - 4\mathrm{i}\,I(1+4I\bar\phi)^{-1}\rho_{0\gamma a}f^{\gamma\alpha}) + (\alpha\leftrightarrow\beta)] = 0\,,
\end{aligned} \tag{4.20}$$

$$\begin{aligned}
&(\delta_{\eta_2}\delta_{\eta_1} - \delta_{\eta_1}\delta_{\eta_2})\bar\psi^{\dot\alpha k} = -4\mathrm{i}\eta^{\beta a}_1\eta_{2\beta a}\partial^{\alpha\dot\alpha}\hat\psi^k_\alpha = 0\,,\\
&(\delta_{\eta_2}\delta_{\eta_1} - \delta_{\eta_1}\delta_{\eta_2})a_{\alpha\dot\alpha} = 4\mathrm{i}\eta^{\beta a}_1\eta_{2\beta a}\partial_{\alpha\dot\alpha}\hat\varphi\,,
\end{aligned} \tag{4.21}$$

where we used the equations of motion derived from the Lagrangian (4.15). The last bracket yields a composite gauge transformation of the gauge field.

Presumably, the transformations (4.18) can be derived from a superfield transformation which is analogous to the one used in the the $N{=}2$ superfield formulation of $N{=}4$ gauge theory in the Minkowski space [19]. We did not elaborate on this point.

5 Deformed charged hypermultiplet

According to eqs. (3.14), (3.13), the superfield action for the case in question reads

$$S_e(V,q) = \tfrac12\int \mathrm{d}u\,\mathrm{d}\zeta^{-4}\ q^+_a\left(D^{++}q^{+\,a} + \frac12\,e[V^{++},q^{+a}]_\star - \frac12\,e(\tau_3)^a_b\{V^{++},q^{+b}\}_\star\right). \tag{5.1}$$

The detailed component structure of this action can be found like in the previous case, applying the general formulas (2.10), (2.11), inserting the $\theta^+,\bar\theta^+$ expansions (2.13), (4.2) and performing integration over the Grassmann and (at the final step) harmonic variables. Solving the harmonic equations for the auxiliary fields, we finally express q^{+a} in terms of off-shell fields of the gauged multiplet and the physical hypermultiplet fields:

$$\begin{aligned}
q^{+a}_e &= u^+_k f^{ak} + \theta^{+\alpha}\rho^a_\alpha + (\theta^+)^2u^-_k g^{ak} + \bar\theta^+_{\dot\alpha}[\chi^{\dot\alpha a} + (\theta^+)^2u^-_k u^-_l\sigma^{\dot\alpha akl}]\\
&+ \theta^+\sigma_m\bar\theta^+ r^{ak}_m u^-_k + (\bar\theta^+)^2[u^-_k h^{ak} + \theta^{+\alpha}u^-_k u^-_l\Sigma^{a\,kl}_\alpha + (\theta^+)^2u^-_k u^-_l u^-_j X^{a\,klj}]\,,
\end{aligned} \tag{5.2}$$

where

$$\begin{aligned}
&g^{ak} = e(\tau_3)^a_b\bar\phi f^{bk}, \quad r^{ak}_m = 2\mathrm{i}(1+2eI\bar\phi)\partial_m f^{ak} + 2e(\tau_3)^a_b A_m f^{bk}\,,\\
&h^{ak} = -4\mathrm{i}eIA_m\partial_m f^{ak} + e(\tau_3)^a_b\left(\phi f^{bk} + 2eI^2\bar\phi\Box f^{bk}\right),\\
&\sigma^{\dot\alpha a\,kl} = 2e(\tau_3)^a_b\bar\Psi^{\dot\alpha(k}f^{bl)}\,, \quad \Sigma^{akl}_\alpha = -4\mathrm{i}eI\bar\Psi^{\dot\alpha(k}\partial_{\alpha\dot\alpha}f^{al)} + 2e(\tau_3)^a_b\Psi^{(k}_\alpha f^{bl)}\,,\\
&X^{aklj} = e(\tau_3)^a_b\mathcal{D}^{(kl}f^{bj)}\,.
\end{aligned} \tag{5.3}$$

The deformed residual U(1) gauge transformations of the charged hypermultiplet components are

$$\begin{aligned}
&\delta_r f^{ak} = \mathrm{i}a\, e(\tau_3)^a_b\, f^{bk}\,, \quad \delta_r \rho^a_\alpha = \mathrm{i}a\, e(\tau_3)^a_b\, \rho^b_\alpha\,,\\
&\delta_r \chi^{\dot\alpha a} = \mathrm{i}a\, e(\tau_3)^a_b\, \chi^{\dot\alpha b} - 2eI\partial^{\alpha\dot\alpha} a\, \rho^a_\alpha\,.
\end{aligned} \tag{5.4}$$

The corresponding component ubroken $N{=}(1,0)$ supersymmetry transformations read

$$\begin{aligned}
&\delta_\epsilon f^{ak} = \epsilon^{\alpha k}\rho^a_\alpha\,, \quad \delta_r \rho^a_\alpha = 2\epsilon^k_\alpha g^a_k = 2\epsilon^k_\alpha\, e(\tau_3)^a_b\, \bar\phi\, f^b_k\,,\\
&\delta_\epsilon \chi^a_{\dot\alpha} = -\epsilon^\alpha_k r^{ak}_{\alpha\dot\alpha} = -2\epsilon^\alpha_k[\mathrm{i}(1+2eI\bar\phi)\partial_{\alpha\dot\alpha} f^{ak} + e(\tau_3)^a_b A_{\alpha\dot\alpha} f^{bk}]\,.
\end{aligned} \tag{5.5}$$

The SW transform for the charged hypermultiplet fields is given by the relations which are similar to those for the 'adjoint' hypermultiplet, eqs. (4.12), though slightly differ in some coefficients

$$\begin{aligned}
&f^{ak}_0 = (1+2eI\bar\phi)f^{ak}\,, \qquad \rho^{\alpha a}_0 = (1+2eI\bar\phi)\rho^{\alpha a}\,,\\
&\chi^a_{\dot\alpha 0} = \chi^a_{\dot\alpha} - 2eI(1+4eI\bar\phi)^{-1}A_{\alpha\dot\alpha}\rho^{\alpha a} + 4eI(1+4eI\bar\phi)^{-1}\bar\Psi_{\dot\alpha k} f^{ak}\,.
\end{aligned} \tag{5.6}$$

The gauge and supersymmetry transformations of fields f^{ak}_0, $\rho^{\alpha a}_0$ and $\chi^a_{\dot\alpha 0}$ look just as the $I=0$ limit of (5.4), (5.5). While checking this, one should take into account that in the considered case of $e \neq 1$ the gauge and supersymmetry transformations of the gauge multiplet fields are obtained by the replacement $I \to eI$ in eqs. (2.14) and (2.15).

The deformed charged hypermultiplet Lagrangian for the physical fields is given by

$$\begin{aligned}
L_e = {} & \tfrac12(1+4eI\bar\phi)\partial_m f_{ak}\partial_m f^{ak} + \mathrm{i}e(\tau_3)^b_a A_m f_{bk}\partial_m f^{ak} + \tfrac12\, e^2(A_m)^2(f^{ak})^2\\
&+ \tfrac12\, e^2\phi\bar\phi(f^{ak})^2 + I^2e^2(f^{ak})^2\Box(\bar\phi^2) - \tfrac12\, e(\tau_3)^a_b f^k_a f^{bl}\mathcal{D}_{kl} + 2\mathrm{i}eI\bar\Psi^{\dot\alpha}_k\rho^\alpha_a\partial_{\alpha\dot\alpha}f^{ak}\\
&+ e(\tau_3)^a_b\Psi^\alpha_k\rho_{\alpha a}f^{bk} + e(\tau_3)^a_b f^k_a\bar\Psi_{\dot\alpha k}\chi^{\dot\alpha b} + \tfrac12\mathrm{i}(1+2eI\bar\phi)\rho^{\alpha a}\partial_{\alpha\dot\alpha}\chi^{\dot\alpha}_a\\
&- \tfrac12\, e(\tau_3)^a_b\rho^\alpha_a A_{\alpha\dot\alpha}\chi^{\dot\alpha b} + \tfrac14\, e(\tau_3)^a_b(\bar\phi\chi_{\dot\alpha a}\chi^{\dot\alpha b} + \phi\rho^\alpha_a\rho^b_\alpha) + \mathrm{i}eI\rho^{\alpha a}A_m\partial_m\rho_{\alpha a}\\
&+ \tfrac12\mathrm{i}\, eI\rho^{\beta a}\rho^\alpha_a\partial_{(\alpha\dot\alpha}A^{\dot\alpha}_{\beta)} + I^2\, e(\tau_3)^a_b\bar\phi\partial_{\alpha\dot\alpha}\rho_{\beta a}\partial^{\beta\dot\alpha}\rho^{\alpha b}\,.
\end{aligned} \tag{5.7}$$

It has to be combined with the $e \neq 1$ modification of the U(1) gauge theory Lagrangian (2.18) rewritten in terms of the original deformed fields. As distinct from the case of neutral hypermultiplet, passing to the undeformed fields in the total action with the help of the transformation (5.6) and the $e \neq 1$ version of (2.17) does not give rise to radical simplifications. Here we present the scalar potential of the model in terms of the deformed fields. It arises as the result of integrating out the gauge multiplet auxiliary field $\mathcal{D}^{kl}$ from the corresponding piece of the total action

$$\tfrac14(1+4eI\bar\phi)^{-2}\mathcal{D}^{kl}\mathcal{D}_{kl} - \tfrac12\, e(\tau_3)^a_b f^k_a f^{bl}\mathcal{D}_{kl} + \tfrac12\, e^2\phi\bar\phi(f^{ak})^2\,, \tag{5.8}$$

and is given by the following positively-definite expression

$$V = \tfrac18\, e^2(f^{ak})^2[(1+4eI\bar\phi)^2(f^{ak})^2 + 4\phi\bar\phi]\,. \tag{5.9}$$

It is worth noting that there is one more mechanism of generating scalar potentials in the considered deformed hypermultiplet-gauge multiplet systems. Namely, one can add to the superfield actions (4.1) or (5.1) the analytic Fayet-Iliopoulos (FI) superfield term

$$S_{FI} = \int \mathrm{d}u\, \mathrm{d}\zeta^{-4}\, \left[3\mathrm{i}c^{++} + c_0(\bar{\theta}^+)^2\right] V^{++}\,, \tag{5.10}$$

where $c^{++} = c^{(ik)}u_i^+ u_k^+\,, \widetilde{c^{++}} = c^{++}$, and $c^{(ik)}$ and c_0 are some harmonic-independent constants (the numerical coefficient 3 was introduced for further convenience). This term is manifestly invariant under $N{=}(1,0)$ supersymmetry and, up to a total harmonic derivative, under gauge transformations (2.12) or (3.6). In WZ gauge for V^{++} FI-term gives rise to the following contribution to the component Lagrangian

$$\mathrm{i}c^{kl}\mathcal{D}_{kl} + c_0\,\bar{\phi}\,, \tag{5.11}$$

which, being combined with (5.8), results in the $\mathrm{SU}(2)_R$-breaking addition to (5.9)

$$\Delta V = \left[e(\tau_3)^a_b(f^k_a f^{bl} c_{kl}) + c^2\right](1 + 4eI\bar{\phi})^2 + c_0\,\bar{\phi}\,. \tag{5.12}$$

It contains the tadpole term $\sim\bar{\phi}$ which could destabilize the theory in the quantum case, giving rise to vacuum transitions. Such a term vanishes under the choice

$$c_0 = -8\,eI\,c^2\,. \tag{5.13}$$

The same mechanism applies to the case of the adjoint hypermultiplet considered in the previous Section. The relevant potential is obtained from (5.12), replacing $eI \to I$ and then setting $e = 0$. Note that such a potential breaks the second (hidden) $N{=}(1,0)$ supersymmetry of this model.

Finally, let us briefly discuss how mass terms for the hypermultiplets can be introduced in the QS-deformed case. The mechanism of such a generation is of the Scherk-Schwarz type and it is similar to the one known in the $N{=}2$ Minkowski case.

Let us consider the constant U(1) analytic potential

$$B^{++} = \bar{m}(\theta^+)^2 + m(\bar{\theta}^+)^2\,, \tag{5.14}$$

where m and $\bar{m}$ are independent real constants (they are mutually conjugated in the Minkowski case). Like in the undeformed case [22, 23], B^{++} is a background solution of the deformed U(1) gauge theory equations. The generation of mass terms can be interpreted as a result of interaction with these constant background 'scalar fields' m amd $\bar{m}$ in (5.14) [23, 24, 25]. It is covenient to define new shifted scalar fields of the gauge U(1) supermultiplet

$$\phi = m + \phi', \qquad \bar{\phi} = \bar{m} + \bar{\phi}'\,. \tag{5.15}$$

The $N{=}(1,0)$ supersymmetry of both gauge field-hypermultiplet models survives after this shift, although its realization on the component fields slightly changes, as can be explicitly seen by substituting (5.15) into (4.11) and (5.5). After making this shift in the Lagrangians (4.5), (5.7) we see that the genuine mass terms appear only for the charged hypermultiplet

and they survive in the commutative limit. Masses of the left- and right-handed fermions ρ^α_a and $\chi^{\dot\alpha}_a$ are proprtional to m and $\bar m$ and so are independent. Also, as a specific feature of the non-anticommutative case, background $\bar m$ induces proper renormalizations of the kinetic terms. The only impact of this background on the Lagrangian of the neutral hypermultiplet is such renormalizations of the kinetic terms.

The new 'free' parts of the superfield actions (4.1) and (5.1) are obtained by substituting there B^{++} for V^{++}, and the shift (5.15) corresponds to decomposing

$$V^{++} = \hat V^{++} + B^{++}\,. \tag{5.16}$$

While the free part of the massless hypermultiplet actions, i.e. (3.1), is invariant under the standard realization of $N{=}(1,0)$ supersymmetry, the free actions with mass terms are invariant under modified supersymmetry transformations which are combinations of the standard $N=(1,0)$ supertranslations and the particular case of U(1) gauge transformations (2.12) (or (3.6) for $e\neq 1$) and (3.2), (3.3) (or (3.7), (3.8)), with

$$\hat\Lambda = -2\bar m \epsilon^- \theta^+\,.$$

E.g., for the charged hypermultiplet the modified $N{=}(1,0)$ transformations read

$$\hat\delta_\epsilon q^+ = (\epsilon^{-\alpha}Q^+_\alpha - \epsilon^{+\alpha}Q^-_\alpha)q^+ - e\,\hat\Lambda \star q^+\,. \tag{5.17}$$

The full off-shell gauge superfield-hypermultiplet action $S_g + S_e(V,q)$ is invariant with respect to (5.17) and the corresponding modification of the $N{=}(1,0)$ transformation of V^{++}

$$\hat\delta_\epsilon V^{++} = (\epsilon^{-\alpha}Q^+_\alpha - \epsilon^{+\alpha}Q^-_\alpha)V^{++} + D^{++}\hat\Lambda + e\,[V^{++},\hat\Lambda]_\star\,. \tag{5.18}$$

The modified transformations close on the constant-parameter subgroup of the corresponding QS-deformed U(1) group, with the appropriate generator being identified with the central charge of the $N{=}(1,0)$ superalgebra. Since $\star$-commutators of a constant parameter Λ with both V^{++} and q^{+a} are vanishing, the modified $N{=}(1,0)$ transformations have a non-zero closure only on the charged hypermultiplet and the anticommutator of the corresponding supercharges is proportional to

$$\sim (\tau_3)^a_b q^{+b}\,,$$

precisely as in the undeformed case. The free massive hypermultiplet actions are invariant just under the modified $N{=}(1,0)$ transformations and by no means under the original ones (which mix the free actions with the interaction terms $\sim \hat V^{++}$).

It is worth pointing out that this modified $N{=}(1,0)$ supersymmetry is present in the theory with the deformation operator constructed out of the standard nilpotent $N{=}(1,0)$ supercharges $Q^\pm_\alpha$ (2.6).

As already mentioned in Section 3, in the case of the complex hypermultiplet there are two mutually commuting rigid U(1) groups. In addition to the transformation (3.10)

on the complex hypermultiplet, one can consider the folowing $\mathrm{SU}(2)_{PG}$-breaking gauge transformation of the same complex superfield:

$$\delta_{\Lambda'} q^{+a} = \tfrac{e}{2} q^{+b} \star \Lambda' [\delta^a_b + (\tau_3)^a_b] \,, \tag{5.19}$$

where Λ' is an independent analytic superfield parameter. This 'right' transformation commutes with the 'left' transformation (3.10). So we are led to introduce two independent deformed U(1) gauge potentials, interacting with the complex hypermultiplet superfield, or e.g. one potential V^{++} for the first gauge group and the constant background potential for the second one. In the second case we shall gain $\mathrm{SU}(2)_{PG}$ breaking mass terms which cannot be generated by shifts of the scalar fields of the 'left' gauge multiplet. Of course we could introduce background fields for the first U(1) group and gauge the second one, then we could generate $\mathrm{SU}(2)_{PG}$ invariant mass terms and 'right' gauge interaction.

Finally we note that the potential (5.12) induced by the FI-term provides another independent $N{=}(1,0)$ supersymmetric source of generating masses for the scalar physical fields of the hypermultiplet (and simultaneously for the field $\bar{\phi}$). The superfield FI-term (5.10) is invariant under both the original and central-charge modified $N{=}(1,0)$ supersymmetries.

6 Conclusions

We have constructed, for the first time, the $N{=}(1,1)$ non-anticommutative singlet Q-deformation (QS-deformation) of a hypermultiplet coupled to a U(1) gauge supermultiplet in Euclidean $\mathbb{R}^4$. We found that there exist two essentially different possibilities of implementing the QS-deformed U(1) gauge action on the hypermultiplet, namely the 'adjoint' and the 'fundamental' representation. These correspond, respectively, to QS-deformations of systems with neutral and charged hypermultiplets. For both options we constructed superfield and component actions, explicitly gave the transformations of unbroken $N{=}(1,0)$ supersymmetry and presented the appropriate Seiberg-Witten transformations to the undeformed fields. The first system was shown to possess an additional (hidden) $N{=}(1,0)$ supersymmetry, thus providing a QS-deformation of Euclidean gauge theory with $N{=}(2,2)$ supersymmetry. We then studied the effect of adding Fayet-Iliopoulos terms to the gauge-superfield-hypermultiplet actions constructed and presented the relevant scalar potentials. We also demonstrated the possibility of generating a mass term for the charged hypermultiplet via the Scherk-Schwarz mechanism related to the appearance of a central charge in the $N{=}(1,0)$ superalgebra. This central charge is identified with the generator of one of the global U(1) symmetries realized on the hypermultiplet.

Our results can be extended along several directions. In particular, it would be interesting to detect possible phenomenological uses of the considered QS-deformations as well as of their non-abelian generalizations. Deformations of this sort may possibly provide a geometrical way of introducing soft supersymmetry breaking. Of course, an appropriate Wick rotation has to be performed to connect with realistic models in Minkowski space. A closely related quantum issue are the renormalization and finiteness properties of our coupled gauge multiplet-hypermultiplet systems, which may be investigated

as in the $N{=}(\frac{1}{2},\frac{1}{2})$ case [7]. The $N{=}(2,2)$ gauge theory is the Euclidean analog of $N{=}4$ super Yang-Mills which supplied the first example of an ultraviolet-finite quantum field theory and displays various other remarkable properties (e.g. in the stringy context of the AdS/CFT correspondence). It is of obvious interest to study the $N{=}(2,2)$ gauge theory from these angles and to examine whether its nilpotent deformation as presented in Section 3 and its non-abelian generalization preserve the basic quantum and geometric properties of the undeformed theory. We recall that nilpotent deformations do not induce noncommutativity for the bosonic spacetime coordinates; hence, such deformed quantum theories are expected not to suffer from the typical non-locality problems such as UV-IR mixing. The issues of their vacuum structure and classical solutions, their non-abelian generalization and the relationship between their Coulomb and Higgs branches all seem to be interesting tasks for future work.

As another possible application of the deformed minimal coupling of a U(1) gauge multiplet with hypermultiplets explicitly worked out in this paper, we mention a generalization of the quotient approach to constructing hyper-Kähler metrics in $N{=}2$ supersymmetric sigma models (see e.g. [26]). Its basic ingredient is a non-propagating $N{=}2$ U(1) gauge multiplet coupled to hypermultiplets. Using the relations given in Sections 3 and 4 it is easy to generalize this approach to QS-deformed $N{=}(1,1)$ theories and to explore the possible impact on the hyper-Kähler target space geometry. Finally, we point out that it is also desirable to elaborate on the implications of more general (non-singlet) Q-deformations in $N{=}(1,1)$ (and perhaps $N{=}(2,2)$) supersymmetric systems with hypermultiplets.

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